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COMPUTER REALIZATION OF SPATIAL FILTERS

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COMPUTER REALIZATION OF SPATIAL FILTERS

by

N.A. Mansour*

ABSTRACT

This report presents theory and results for a computer technique used to construct optical spatial filters. The technique only considers the change in phase and the filter constructed therefore is a phase filter. This filter can be used to improve the beating efficiency which in turn improves the signal to noise ratio in the well known heterodyne optical detection technique. Mathematical formulation of the technique is presented and a 32 grey scale was used to realize the filters for a letter O.

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FABRICATION ASSISTÉE PAR ORDINATEUR DE FILTRES OPTIQUES SPATIAUX

par

N.A. Mansour*

RÉSUMÉ

Dans ce rapport, l'auteur présente la théorie sur laquelle se fonde une technique automatisée utilisée pour fabriquer des filtres optiques spatiaux et les résultats obtenus. La technique repose uniquement sur le changement de phase et, par conséquent, le filtre qu'elle permet de mettre au point est un filtre de déphasage qui peut servir à améliorer l'efficacité du battement, ce qui, en retour, améliore le rapport signal-sur-bruit de la technique bien connue de détection optique hétérodyne. La formule mathématique de la technique est présentée et une échelle de gris 32 a été utilisée pour la mise au point de filtres pour une lettre O.

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INTRODUCTION

The kinoform is a computer-generated element which can be physically visualized as a wavefront reconstruction device. It operates only on the phase of the incident wave to form a single image of an object.

The kinoform was originally invented at IBM in 1969 by Lesem, Hirsch and Jordan (1). These workers were able to obtain a recognizable image by constructing a kinoform using a random phase generator. However, they gave no account of the type and limits of the random phase they utilized to obtain the image. Commenting on their paper, Katyl stated "Although this method can produce phase functions $\phi(r_k)$ which reconstruct images that are good approximations to the original object, the method has the disadvantage that physical guidelines are absent in the search for the best phase distribution" (2). This observation along with the view expressed by Fillmore, "Although the kinoform was introduced in 1969 and has received considerable attention since that time, it is still not very well understood", make research in this area more empirical than rational (3). The only work reported on the kinoform after its introduction has been that of Katyl (1972) and Fillmore (1972) in which they constructed kinoforms of one-dimensional objects. However, it is not clear how such studies can be utilized to yield the required information about the random phase distribution in two dimensions to construct the corresponding kinoform.

Our interest in kinoform synthesis and realization arose from its ability to change the phase distribution of a certain wave. In our laboratory, this technique could be utilized for image enhancement and for improving the signal to noise ratio in laser scattering experiments. Thus to obtain information on the random phase distribution at the object plane, we decided, as a convenient starting point, to try to construct kinoforms of symmetrical objects.

METHODOLOGY

KINOFORM CONSTRUCTION

Kinoforms are made by calculating the wavefront scattered from the numerically defined object. Here, only the distribution of phase needs to

be considered and it should be reduced to fall in the range 0 to 2π. A photographic plot of the phase distribution (mod 2π) is made and converted into a phase plot. This plot is achieved by considering that a phase of 2π will correspond to a completely black area and a phase of zero will correspond to a completely clear area on a photographic film bleached in such a way that it possesses a type of surface relief. For example, the portion of the film which is completely black i.e., phase 2π, will have twice the thickness of the portion having phase π. Exposing the bleached film to a laser beam, considered to be a plane wave with amplitude A, will impress the phase with the resultant wave thus taking the form $Ae^{j\phi(x,y)}$.

Because of the lack of information on either theoretical or experimental approaches for choosing the appropriate phases for this study, we started by constructing the kinoforms of symmetrical objects in the form of a computer-constructed square aperture and the letter O. The following options were chosen:

1. no phases at all.
2. phases taken at random with values lying in the range 0 to π.
3. phases taken at random with values lying in the range 0 to 0.5.
4. phases of 0 or 1 taken at random.

REALIZATION OF KINOFORMS

MATHEMATICAL MODEL

To realize the kinoform mathematically, the object can be imagined as being a two-dimensional array of point apertures. Each aperture is assigned a transmittance value T expressed as

$$T_{ij} = t_{ij} \exp[i\alpha_{ij}]$$

The values of t_{ij} represent the transmissivity of the aperture and range from zero for an opaque aperture, to unity for a completely transparent aperture. The values of α_{ij} model the diffuse scattering from the objects, i.e., the phase changes of the wavefront at the objects and are obtained from a random number generator.

A one-dimensional example will illustrate the computations required to prepare a scattered wave pattern. If the area of the object is small

compared with the distance Z_o from the object to the kinoform plane, Kirchhoff diffraction theory provides an expression for the field from the single aperture:

$$W_j(x) \alpha \exp[ik(x-a_j)^2] \qquad \text{Eq 1}$$

where,

$$k = \frac{\pi}{\lambda Z_o},$$

a_j is the coordinate of the jth aperture and λ is the wavelength. For an even number "m" of equally-spaced apertures on the grid we set $a_j = j \Delta a$ and obtain the resultant field by summation:

$$W(x) = \sum_{j=\frac{-m}{2}}^{\frac{m}{2}-1} W_j(x) = \exp(ikx)^2 \sum_{j=\frac{-m}{2}}^{\frac{m}{2}-1} U_j \exp\left(-\frac{2\pi i x j \Delta a}{\lambda Z_o}\right) \qquad \text{Eq 2}$$

where, $T_j = T_j \exp[ik(j \Delta a)^2]$.

To evaluate W(x) at equally-spaced points on a grid at the kinoform plane we let $x \rightarrow x_\ell = \ell \Delta x$. The dimensions on Δx and Δa have to be chosen to satisfy the condition,

$$\Delta x \Delta a = \frac{\lambda Z_j}{m}$$

where $Z_j = Z_o$ in the one-dimensional case. This condition permits replacing the sum in Eq 2 by a Finite Fourier transform which can be calculated using the Cooley-Tukey algorithm (4). Equation 2 can then be written as

$$W(x_\ell) = \exp[ik(\ell \Delta x)^2] \sum U_j \exp\left[-2\pi i \left(\frac{\Delta x \Delta a}{\lambda Z_o}\right) \ell j\right].$$

Taking $\frac{\Delta x \Delta a}{\lambda Z_o} = \frac{1}{m}$,

$$W(x_\ell) = \exp[ik(\ell \Delta x)^2] \sum_{j=\frac{-m}{2}}^{\frac{m}{2}-1} W_j \exp\left(-2\pi i \frac{\ell j}{m}\right)$$

$$= U(x_\ell) \exp [ik(\ell \Delta x)^2] \qquad \text{Eq 3}$$

where, $U(x_\ell)$ is the Finite Fourier transform of W_j. For the two-dimensional case, the field will be given by

$$W(x_{\ell 1}, x_{\ell 2}) = \exp [ik\{(\ell_1 \Delta x)^2 + (\ell_2 \Delta x)^2\}]$$

$$\sum_{j_1=\frac{-m}{2}}^{\frac{m}{2}-1} \sum_{j_2=\frac{-m}{2}}^{\frac{m}{2}-1} U_{j_1 j_2} \exp \left[-2\pi i \left(\frac{\ell_1 j_1}{m} + \frac{\ell_2 j_2}{m} \right)\right] \qquad \text{Eq 4}$$

where, $U_{j_1 j_2} = T_{j_1 j_2} \exp \{ ik[j_1 \Delta a_1)^2 + (j_2 \Delta a_2)^2]\}$ Eq 5

Experimental Technique

The kinoform is realized using a 4-step procedure involving calculations and photographic techniques. The first step consists of calculating the field i.e., the phase distribution, and reducing it to come within the range of 0 to 2π. In the second step, these data are recorded on a photographic plate, bearing in mind that 2π corresponds to completely black areas and zero to completely clear areas. The third and fourth steps involve photographic reduction of the recorded kinoform to sizes that are appropriate for illumination with the visible light chosen then bleaching of the photographic plate. The bleaching step effects removal of the blackness leaving a surface relief on the film; those areas of the film which have a phase 2π will have double the thickness of those areas which have a phase π.

Phase Plot

After calculating the phase distribution of the defined object based on the appropriate mathematical model (Equation 4) mentioned above, the phase is reduced to come in the range 0 to 2π and is plotted as a gradation between completely black and completely white areas. This gradation can be accomplished by creating a grey scale taking, for example 32 grey levels between completely black and completely white areas. Lessem, et al plotted the phase of the scattered object wave photographically on a Geospace Corporation Dp 203 (IBM 1780) plotter using a 32-level grey-scale (1).

In our studies we used two methods of realizing the phases. The first method utilized an IBM 360 line printer in an overstriking mode while the second method involved controlling the output light of a cathode ray tube so as to create a maximum of 128 dots in a given area of a photographic plate. The line printer method is the same as that used for simulating electron micrographs (5,6). Villagrana and white used a 32-level grey scale in their work but did not mention the intensity of each level; the intensity of the grey scale is dependent on the characteristics of the line printer used (6). Figure 1 shows the grey scale created by the IBM 360 line printer. The grey scale in the second method was obtained by changing the number of dots per unit area. Figure 2 shows a 64-level grey scale created using the light controlled plotter made available for this study through the generosity of Imperial Oil at Calgary.

Bleaching

The bleaching technique used in this study was the same as the one used for making phase holograms. To obtain a high resolution phase hologram, the bleaching process must effect replacement of the developed silver in the absorption hologram emulsion by a transparent silver salt whose refractive index is generally greater than that of the gelatin. When this is achieved, scattering of incident light is substantially greater from the salt than from the silver grains in the unbleached hologram.

Bleach baths for converting the silver to a transparent silver salt usually contain one of the following oxidizing agents: mercuric chloride, cupric chloride (7), ferric chloride, potassium ferricyanide (8) or potassium bromide (9). Oxidation of silver $Ag \rightarrow Ag^{+}$ is accomplished through a chemical reaction involving reduction of a multivalent metallic ion from its higher valence state to a lower one. For example, $Hg^{2+} \rightarrow Hg^{1+}$ and $Fe^{3+} \rightarrow Fe^{2+}$. The resulting silver salt is usually a halide such as AgCl or AgBr.

Two bleaching methods were used in this study, one suggested by the Ilford Company and the other based on using Kodak ER-2 (9). The Ilford method gave the best results and involved immersing the film or plate after washing and draining into a bath consisting of a mixture of two solutions. The solutions were prepared as follows and mixed in the ratio 9 parts solution A to one part solution B.

Solution A

cupric chloride	10 gm
acetic acid, glacial	50 cm^3
water	1000 cm^3

Solution B

Hydrogen peroxide	20 volumes

The recipe used by Pennington and Harper was that of a typical halide bleach (Kodak EB-2), mixed in the ratio 1 part A to 1 part B just prior to use:

Part A

Cupric sulphate, crystal	120 gm
citric acid	150 gm
potassium bromide	7.5 gm
water	1 litre

Part B

Hydrogen peroxide	3%

Some authors have also noted that development using Kodak D76 leads to less noise in the bleached holograms than does standard development using Kodak D19 and that the efficiency and noise of the hologram are critically dependent on the drying procedure.

RESULTS AND DISCUSSION

As mentioned earlier two symmetric objects, a square aperture and the letter "O" were chosen to exploer the details of the kinoform technique.

Each object was constructed as an array of points. Thus, the kinoform phase transmission function was determined using a computer program based on the Fast Fourier algorithm (4). In the two examples a 64 x 64 matrix was used. Recalling equations 4 and 5

$$W(x_{\ell 1}, x_{\ell 2}) = \exp ik[(\ell_1 \Delta ax)^2 + (\ell_2 \Delta y)^2] F[t_{j_1 j_2} \exp i\alpha_{ij} \exp iK[(j_1 \Delta a_1)^2 + (j_2 \Delta a_2)^2]]$$

$$= \exp ik[(\ell_1 \Delta x)^2 + (\ell_2 \Delta y)^2] \; F\,[U_{j_1 j_2}]$$

$$= \exp (i\phi_1) \; F\,[t_{j_1 j_2} \exp i\alpha_{j_1 j_2} \exp i\theta] \qquad \text{Eq 5}$$

where $F[U_{j_1 j_2}]$ is the Fourier transform of $U_{j_1 j_2}$, Δx and Δy = the distance between the sampling points in the x and y directions in the kinoform plane and Δa_1 and Δa_2 = the distances in the x and y directions in the object plane.

The sum over the function $U_{j_1 j_2}$ was done so that the phase $k[(j_1 \Delta a_1)^2 + (j_2 \Delta a_2)^2]$ was zero at the origin of the object and a symmetric phase taken on both sides of the object. Here $\alpha_{j_1 j_2}$ is the random phase which has to be choosen to get the best phase distribution assigned to the object points to give a recognizable image. If $\alpha_{j_1 j_2}$ is taken as zero, the input to the Fourier transform will be a symmetric function and consequently, the phase plot (kinoform phase) will also be symmetric. In our calculations the values used were:

$k = \frac{\pi}{\lambda Z_0}$ with λ=0.00007 cm and Z_0=100 cm, $\Delta x = \Delta y = 7/640$ cm and $\Delta a_1 = \Delta a_2 = 0.01$ cm.

Figure 3 shows the phase of the Fourier transform of the square aperture for $\phi = \alpha_{j_1 j_2} = \theta = \theta$ and transmittance, $t_{j_1 j_2} = 1$ at the aperture and = θ elsewhere. It can be seen that for these conditions, the figure was symmetric. If for this square aperture, on the other hand, only $\alpha_{j_1 j_2}$ is taken = 0.0, we will be left with a symmetric input function to the Fourier transform. However, it is important to note that the input function to the Fourier transform is symmetric in a square manner $K[(j_1 \Delta a_1)^2 + (j_2 \Delta a_2)^2]$. The result obtained in this case was that shown in Fig. 4 using the light pen method or in Fig. 5., plotted using the overstriking method. One could be misled into thinking that the square aperture would yield a distribution of phase where no rounding-off effects near the corners would be seen as in Fig. 4. In fact, this rounding, which results from the squaring happens to be in the phase of equation 5.

The reason for arriving at such a conclusion arises from the phase distribution shown in Fig. 6 where the value of B was taken as $k[j_1 \Delta a_1 + j_2 \Delta a_2]$ i.e., ignoring the squaring in $(j_1 \Delta a_1)$ and

$(j_2 \Delta a_2)$ and using the overstriking plotting method. The shape of a square aperture is retained since there is a linear relation between the phases at the different points of the object. However, note as shown in Fig. 6 that the square aperture looks like a rectangle when using the overstriking method of plotting the phases. The line printer prints 10 characters per inch, but only 6 lines per inch.

The next step was to take into account the random phases denoted by $\alpha_{j_1 j_2}$ in equation 6 for a square aperture. Figure 7 shows the results obtained where the random phase $\alpha_{j_1 j_2}$ had values taken at random between 0 and 0.5. Figures 8 and 9 respectively show the results obtained when $\alpha_{j_1 j_2}$ was in the range between 0 and 3.14 at random, and when the random phase was taken to be either 0 or 1 at random.

These calculations were repeated where the object was the letter O. The results obtained for $\alpha_{j_1 j_2} = 0$, $\alpha_{j_1 j_2}$ taken at random in the range 0 up to 3.14 and $\alpha_{j_1 j_2} = 0$ or 1 at random, are given in Fig. 10, 11 and 12 respectively. Where the random phase $\alpha_{j_1 j_2}$ does not equal 0, it is evident that the phase distribution in the kinoform plane is strongly dependent on the random phase distribution $\alpha_{j_1 j_2}$ chosen for the object points. (Fig. 10 - 12). This conclusion has an important practical application in image enhancement. That is to say, by changing phase distribution in the kinform plane, it is possible to generate a phase filter which can improve the quality of a given image.

Being large, the size of the computed wavefront phase was not appropriate for creating the final image of the object. Thus the original display had to be reduced to a size where the distance between the sample points was comparable to the wavelength of visible light. Reduction was done using a Leitz Wetzler FOCOTAR-f4.5-50 mm lens. Since the reproducibility of the original picture depends on the type of film and on development as well as the lens used in the reduction, it was necessary to experiment with different reduction procedures until a reasonable standardized procedure was found.

Again, experimentation with variations of the bleaching procedures mentioned earlier revealed that the Ilford method usually gave a better image when the bleached photograph was exposed to the laser beam.

CONCLUSIONS

This study demonstrated that the kinoform could be readily adapted for use as a viable technique for image enhancement. However, with further study would be needed for its use in improving the s/n ratio in laser scattering experiments. For this technique to be adopted in the above applications, it is suggested to interested researchers to carry out the following:

1. to study the effect of various phase distributions in improving s/n ratio in these types of experiments,

2. to study the effect of various bleaching techniques in order to determine the optimum method.

REFERENCES

1. Lesem, L.B., Hirsch, P.M., Jordan, J.A.; "The kinoform: A new wavefront reconstruction device"; IBM J. Res. Develop. 13 (1969) 150.

2. Katyl, R.H.; "Use of pseudo random sequences in the synthesis of kinoforms"; Appl. Opt. 11(1972)198.

3. Filmore, G.L.; "Kinofrom viewed as a random number generator"; Appl. Opt. 11(1972)2193.

4. Cooley, J.W. and Tukey, J.W.; "An algorithm for the machine calculation of complex fourier series"; Mathematics of Computation 19(1967)297.

5. Head, A.K.; "The computer generation of electron microscope pictures of dislocations;" Aust. J. Phys. 20(1967)557.

6. Villagrana, R.E., White, P.H.; "Computer generated electron diffraction patterns"; Septieme congres International De Microscopie Electronique; Aug. 30-Sept. 5; Grenoble (1970)205.

7. Ilford Technical Information book.

8. Burckhardt, C.B. and Doherty, E.T.; "A bleach process for high efficiency low noise holograms"; Appl. Opt. 8(1969)2479.

9. Pennington, K.S. and Harper, J.S.; "Techniques for producing low noise improved efficiency holograms"; Appl. Opt. 9(1970)1643.

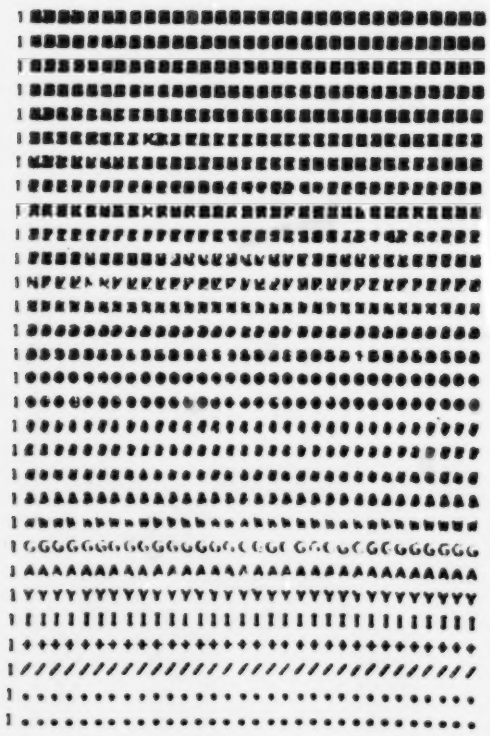

Fig. 1 photograph of the 32 level grey scale using the overstriking method: IBM 360 line printer.

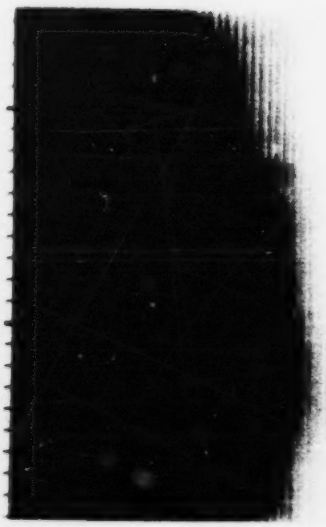

Fig. 2 photograph of the 64 level grey scale using computer controlled light pen.

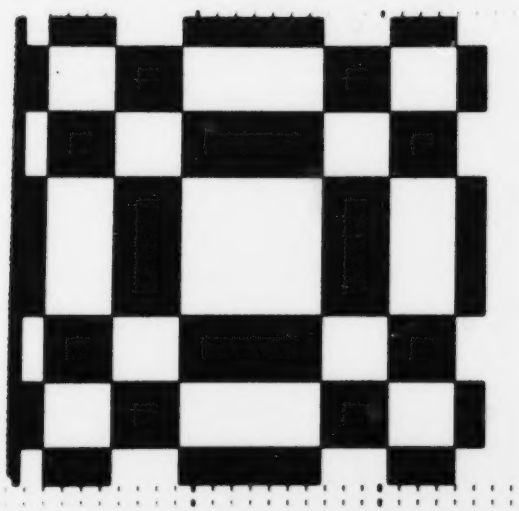

Fig. 3 Fourier Transform of a square aperture for transmittance = 1 at the aperture and zero otherwise.

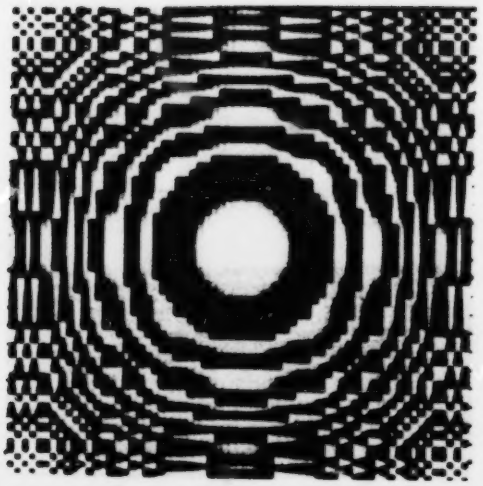

Fig. 4 Computed wave front phase of square aperture where the random phases α_{j1j2} =o, using computer controlled light pen.

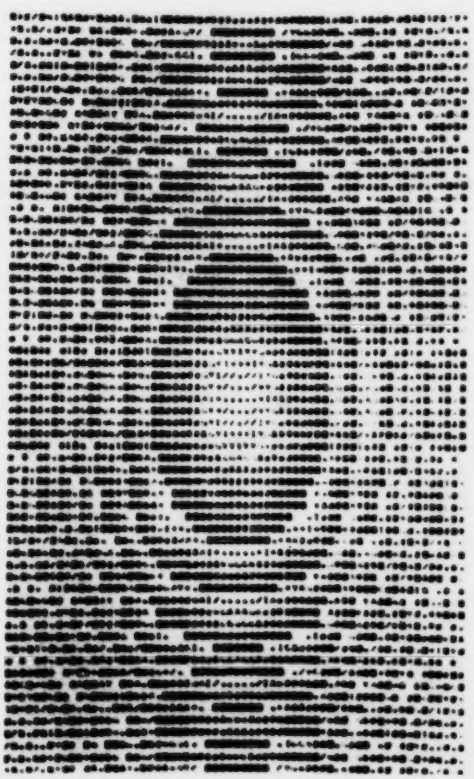

Fig.5 Computed wave front phase of a sqaure aperture for the random α_{j1j2} = 0, using the overstriking plotting technique.

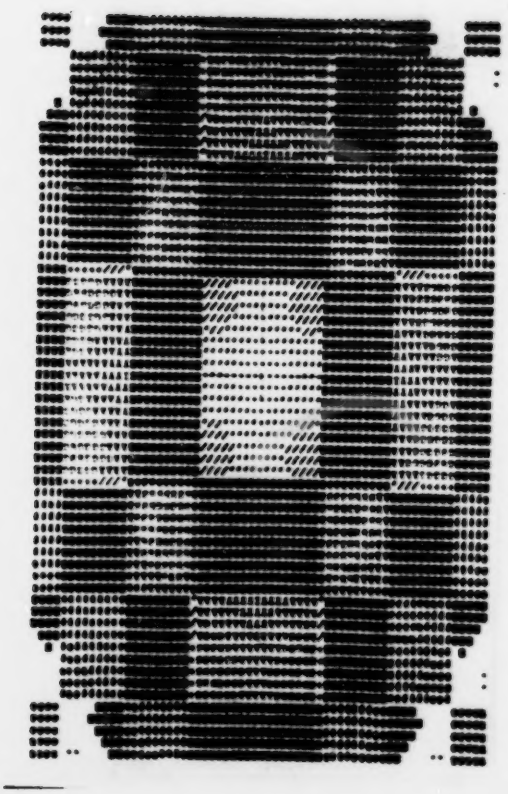

Fig. 6 The computed wave front phase of a square aperture taking the input phases to the Fourier Transform in a linear relation and $\alpha_{j1j2} = 0$, using the overstriking plotting technique.

Fig. 7 The computed wave front phase of a square aperture taking α_{j1j2} at random in the range zero up to 0.5.

Fig. 8 Computed wave front phase of a square aperture taking α_{j1j2} at random in the range zero up to 3.14.

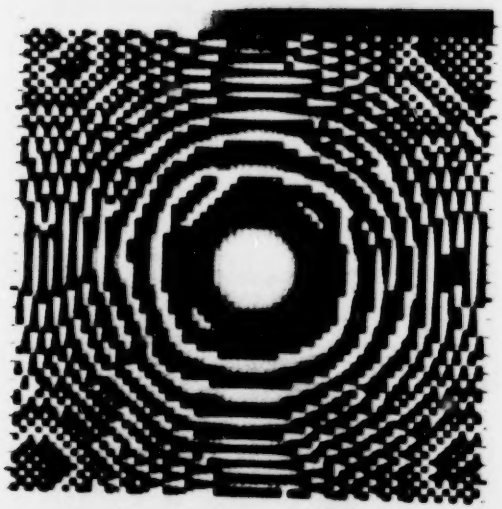

Fig. 9 The computed wave front phase of a square aperture taking α_{j1j2} to be zero or 1 at random.

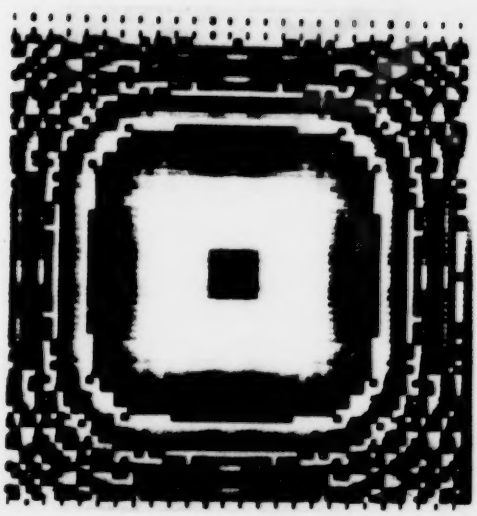

Fig. 10 The computed wave front phase of a letter O taking $\alpha_{j1j2} = 0$.

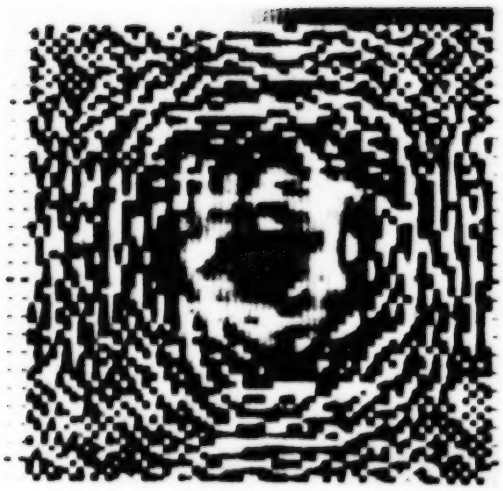

Fig. 11 The computed wave front phase of a letter O taking α_{j1j2} to be zero or 3.14 at random.

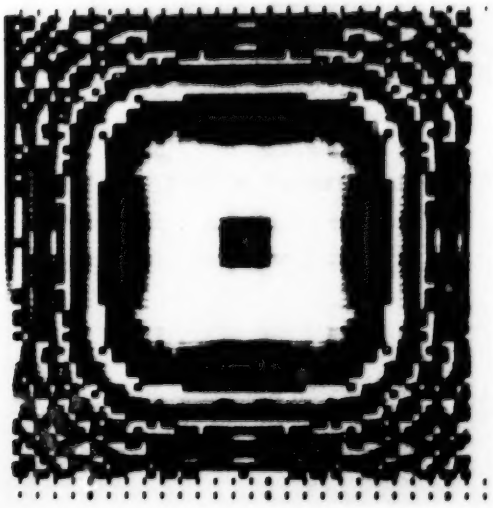

Fig. 12 The computed wave front phase of a letter O taking α_{j1j2} to be zero or 1 at random.